Experimental Probability

COMMON CORE

? ESSENTIAL QUESTION

How can you use experimental probability to solve real-world problems?

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Real-World Video

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Meteorologists use sophisticated equipment to gather data about the weather. Then they use experimental probability to forecast, or predict, what the weather conditions will be.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.

Simplify Fractions

EXAMPLE Simplify $\frac{12}{21}$.

12: 1, 2, ③, 4, 6, 12   List all the factors of the numerator and denominator.
21: 1, ③, 7, 21   Circle the greatest common factor (GCF).

$\frac{12 \div 3}{21 \div 3} = \frac{4}{7}$ Divide the numerator and denominator by the GCF.

Write each fraction in simplest form.

1. $\frac{6}{10}$ _____

2. $\frac{9}{15}$ _____

3. $\frac{16}{24}$ _____

4. $\frac{9}{36}$ _____

5. $\frac{45}{54}$ _____

6. $\frac{30}{42}$ _____

7. $\frac{36}{60}$ _____

8. $\frac{14}{42}$ _____

Write Fractions as Decimals

EXAMPLE

$\frac{13}{25} \rightarrow$

$$\begin{array}{r} 0.52 \\ 25\overline{)13.00} \\ -12.5 \\ \hline 50 \\ -50 \\ \hline 0 \end{array}$$

Write the fraction as a division problem.
Write a decimal point and a zero in the dividend.
Place a decimal point in the quotient.
Write more zeros in the dividend if necessary.

Write each fraction as a decimal.

9. $\frac{3}{4}$ _____

10. $\frac{7}{8}$ _____

11. $\frac{3}{20}$ _____

12. $\frac{19}{50}$ _____

Percents and Decimals

EXAMPLE

$109\% = 100\% + 9\%$
$= \frac{100}{100} + \frac{9}{100}$
$= 1 + 0.09$
$= 1.09$

Write the percent as the sum of 1 whole and a percent remainder.
Write the percents as fractions.
Write the fractions as decimals.
Simplify.

Write each percent as a decimal.
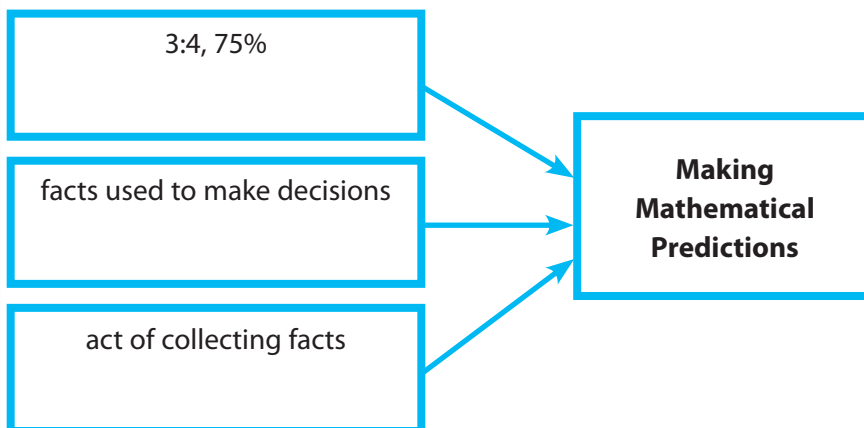
13. 67% _____

14. 31% _____

15. 7% _____

16. 146% _____

Write each decimal as a percent.

17. 0.13 _____

18. 0.55 _____

19. 0.08 _____

20. 1.16 _____

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic. You can put more than one word in each box.

```
┌─────────────────────────┐
│        3:4, 75%         │───┐
└─────────────────────────┘   │
                              ↘
┌─────────────────────────┐     ┌──────────────┐
│ facts used to make      │───→ │   Making     │
│ decisions               │     │ Mathematical │
└─────────────────────────┘     │ Predictions  │
                              ↗  └──────────────┘
┌─────────────────────────┐   │
│ act of collecting facts │───┘
└─────────────────────────┘
```

Understand Vocabulary

Match the term on the left to the definition on the right.

1. probability   **A.** Measures the likelihood that the event will occur.

2. trial   **B.** A set of one or more outcomes.

**3.** event   **C.** Each observation of an experiment.

Vocabulary

Review Words
✔ data *(datos)*
✔ observation *(observación)*
✔ percent *(porcentaje)*
✔ ratio *(razón)*

Preview Words
complement *(complemento)*
compound event *(suceso compuesto)*
event *(suceso)*
experiment *(experimento)*
experimental probability *(probabilidad experimental)*
outcome *(resultado)*
probability *(probabilidad)*
simple event *(suceso simple)*
simulation *(simulación)*
trial *(prueba)*

Active Reading

Pyramid Before beginning the module, create a rectangular pyramid to help you organize what you learn. Label each side with one of the lesson titles from this module. As you study each lesson, write important ideas, such as vocabulary, properties, and formulas, on the appropriate side.
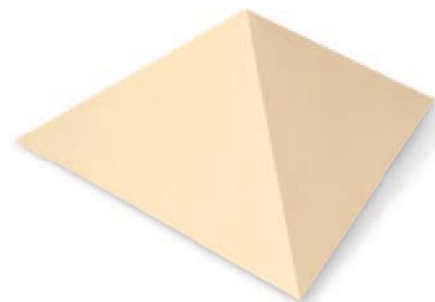
Experimental Probability

COMMON CORE

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE **7.SP.6**

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Key Vocabulary

simple event *(suceso simple)*
An event consisting of only one outcome.

experimental probability *(probabilidad experimental)*
The ratio of the number of times an event occurs to the total number of trials, or times that the activity is performed.

What It Means to You

You will use experimental probabilities to make predictions and solve problems.

EXAMPLE 7.SP.6

Caitlyn finds that the experimental probability of her making a goal in hockey is 30%. Out of 500 attempts to make a goal, about how many could she predict she would make?

$$\frac{3}{10} \cdot 500 = x$$

$$150 = x$$

Caitlyn can predict that she will make about 150 of the 500 goals that she attempts.

COMMON CORE **7.SP.7b**

Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

Key Vocabulary

sample space *(espacio muestral)*
All possible outcomes of an experiment.

What It Means to You

You will use data to determine experimental probabilities.

EXAMPLE 7.SP.7b

Anders buys a novelty coin that is weighted more heavily on one side. He flips the coin 60 times and a head comes up 36 times. Based on his results, what is the experimental probability of flipping a head?

$$\text{experimental probability} = \frac{\text{number of times event occurs}}{\text{total number of trials}}$$

$$= \frac{36}{60} = \frac{3}{5}$$

The experimental probability of flipping a head is $\frac{3}{5}$.

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12.1 Probability

? ESSENTIAL QUESTION

How can you describe the likelihood of an event?

EXPLORE ACTIVITY   **COMMON CORE** 7.SP.5

Finding the Likelihood of an Event

Each time you roll a number cube, a number from 1 to 6 lands face up. This is called an *event*.

Work with a partner to decide how many of the six possible results of rolling a number cube match the described event.

Then order the events from least likely (1) to most likely (9) by writing a number in each box to the right.

Rolling a number less than 7 _____ ☐

Rolling an 8 _____ ☐

Rolling a number greater than 4 _____ ☐

Rolling a 5 _____ ☐

Rolling a number other than 6 _____ ☐

Rolling an even number _____ ☐

Rolling a number less than 5 _____ ☐

Rolling an odd number _____ ☐

Rolling a number divisible by 3 _____ ☐

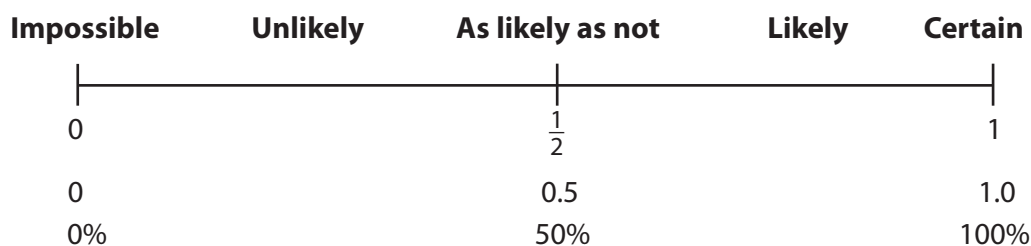### Reflect

1. Are any of the events impossible? _____

_____

Describing Events

An **experiment** is an activity involving chance in which results are observed. Each observation of an experiment is a **trial**, and each result is an **outcome**. A set of one or more outcomes is an **event**.

The **probability** of an event, written *P*(event), measures the likelihood that the event will occur. Probability is a measure between 0 and 1 as shown on the number line, and can be written as a fraction, a decimal, or a percent.

If the event is not likely to occur, the probability of the event is close to 0. If an event is likely to occur, the event's probability is closer to 1.

Impossible	Unlikely	As likely as not	Likely	Certain

$$0 \qquad\qquad\qquad\qquad \frac{1}{2} \qquad\qquad\qquad\qquad 1$$

0	0.5	1.0
0%	50%	100%

EXAMPLE 1   Real World

COMMON CORE   7.SP.5

Tell whether each event is impossible, unlikely, as likely as not, likely, or certain. Then, tell whether the probability is 0, close to 0, $\frac{1}{2}$, close to 1, or 1.

A You roll a six-sided number cube and the number is 1 or greater.

> *Because you can roll the numbers 1, 2, 3, 4, 5, and 6 on a number cube, there are 6 possible outcomes.*

This event is certain to happen. Its probability is 1.

B You roll two number cubes and the sum of the numbers is 3.

This event is unlikely to happen. Its probability is close to 0.

C A bowl contains disks marked with the numbers 1 through 10. You close your eyes and select a disk at random. You pick an odd number.

This event is as likely as not. The probability is $\frac{1}{2}$.

D A spinner has 8 equal sections marked 0 through 7. You spin and land on a prime number.

> *Remember that a prime number is a whole number greater than 1 and has exactly 2 divisors, 1 and itself.*

This event is as likely as not. The probability is $\frac{1}{2}$.

Math Talk

Mathematical Practices

Is an event that is *not* certain an impossible event? Explain.

Reflect

2. The probability of event *A* is $\frac{1}{3}$. The probability of event *B* is $\frac{1}{4}$. What can you conclude about the two events?

_____

_____

3. A hat contains pieces of paper marked with the numbers 1 through 16. Tell whether picking an even number is impossible, unlikely, as likely as not, likely, or certain. Tell whether the probability is 0, close to 0, $\frac{1}{2}$, close to 1, or 1.

_____

Finding Probability

The **sample space** is a set of all possible outcomes for an event. A sample space can be small, such as the 2 outcomes when a coin is flipped. Or a sample space can be large, such as the possible number of Texas Classic automobile license plates. Identifying the sample space can help you calculate the probability of an event.

Probability of An Event

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of equally likely possible outcomes}}$$

EXAMPLE 2   Real World

COMMON CORE   **7.SP.7a**

What is the probability of rolling an even number on a standard number cube?

STEP 1   Find the sample space for a standard number cube.

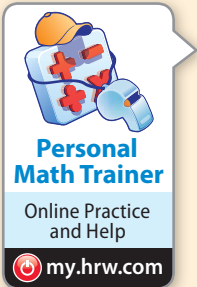{1, 2, 3, 4, 5, 6}   *There are 6 possible outcomes.*

STEP 2   Find the number of ways to roll an even number.

2, 4, 6   *The event can occur 3 ways.*

STEP 3   Find the probability of rolling an even number.

$$P(\text{even}) = \frac{\text{number of ways to roll an even number}}{\text{number of faces on a number cube}}$$

$$= \frac{3}{6} = \frac{1}{2}$$ *Substitute values and simplify.*

The probability of rolling an even number is $\frac{1}{2}$.

YOUR TURN

Find each probability. Write your answer in simplest form.

4. Picking a purple marble from a jar with 10 green and 10 purple marbles. _____

5. Rolling a number greater than 4 on a standard number cube.

_____

Using the Complement of an Event

The **complement** of an event is the set of all outcomes in the sample space that are *not* included in the event. For example, in the event of rolling a 3 on a number cube, the complement is rolling any number other than 3, which means the complement is rolling a 1, 2, 4, 5, or 6.

An Event and Its Complement

The sum of the probabilities of an event and its complement equals 1.

$$P(\text{event}) + P(\text{complement}) = 1$$

You can apply probabilities to situations involving random selection, such as drawing a card out of a shuffled deck or pulling a marble out of a closed bag.

EXAMPLE 3 🌎 Real World   COMMON CORE 7.SP.7a

There are 2 red jacks in a standard deck of 52 cards. What is the probability of not getting a red jack if you select one card at random?

$$P(\text{event}) + P(\text{complement}) = 1$$

$$P(\text{red jack}) + P(\text{not a red jack}) = 1 \qquad \text{The probability of getting a red jack is } \tfrac{2}{52}.$$

$$\tfrac{2}{52} + P(\text{not a red jack}) = 1 \qquad \text{Substitute } \tfrac{2}{52} \text{ for } P(\text{red jack}).$$

$$\tfrac{2}{52} + P(\text{not a red jack}) = \tfrac{52}{52} \qquad \text{Subtract } \tfrac{2}{52} \text{ from both sides.}$$

$$\underline{-\tfrac{2}{52} \qquad\qquad\qquad -\tfrac{2}{52}}$$

$$P(\text{not a red jack}) = \tfrac{50}{52}$$

$$P(\text{not a red jack}) = \tfrac{25}{26} \qquad \text{Simplify.}$$

The probability that you will not draw a red jack is $\tfrac{25}{26}$. It is likely that you will not select a red jack.

Reflect

6. Why do the probability of an event and the probability of its complement add up to 1?

_____

_____

_____

_____

_____

YOUR TURN

7. A jar contains 8 marbles marked with the numbers 1 through 8. You pick a marble at random. What is the probability of not picking the marble marked with the number 5? _____

8. You roll a standard number cube. Use the probability of rolling an even number to find the probability of rolling an odd number. _____
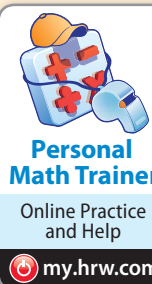
Guided Practice

1. In a hat, you have index cards with the numbers 1 through 10 written on them. Order the events from least likely to happen (1) to most likely to happen (8) when you pick one card at random. In the boxes, write a number from 1 to 8 to order the eight different events. (Explore Activity)

You pick a number greater than 0.   ☐

You pick an even number.   ☐

You pick a number that is at least 2.   ☐

You pick a number that is at most 0.   ☐

You pick a number divisible by 3.   ☐

You pick a number divisible by 5.   ☐

You pick a prime number.   ☐

You pick a number less than the greatest prime number.   ☐

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Determine whether each event is impossible, unlikely, as likely as not, likely, or certain. Then, tell whether the probability is 0, close to 0, $\frac{1}{2}$, close to 1, or 1. (Example 1)

2. randomly picking a green card from a standard deck of playing cards

_____

3. randomly picking a red card from a standard deck of playing cards

_____

4. picking a number less than 15 from a jar with papers labeled from 1 to 12

_____

5. picking a number that is divisible by 5 from a jar with papers labeled from 1 to 12

_____

Find each probability. Write your answer in simplest form. (Example 2)

6. spinning a spinner that has 5 equal sections marked 1 through 5 and landing on an even number

_____

7. picking a diamond from a standard deck of playing cards which has 13 cards in each of four suits: spades, hearts, diamonds and clubs

_____

Use the complement to find each probability. (Example 3)

8. What is the probability of not rolling a 5 on a standard number cube?

_____

9. A spinner has 3 equal sections that are red, white, and blue. What is the probability of not landing on blue?

_____

10. A spinner has 5 equal sections marked 1 through 5. What is the probability of not landing on 4?

_____

11. There are 4 queens in a standard deck of 52 cards. You pick one card at random. What is the probability of not picking a queen?

_____

? **ESSENTIAL QUESTION CHECK-IN**

12. Describe an event that has a probability of 0% and an event that has a probability of 100%.

_____

_____

_____

12.1 Independent Practice

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13. There are 4 aces and 4 kings in a standard deck of 52 cards. You pick one card at random. What is the probability of selecting an ace or a king? Explain your reasoning.

_____

_____

14. There are 12 pieces of fruit in a bowl. Seven of the pieces are apples and two are peaches. What is the probability that a randomly selected piece of fruit will not be an apple or a peach? Justify your answer.

_____

_____

15. Critique Reasoning For breakfast, Clarissa can choose from oatmeal, cereal, French toast, or scrambled eggs. She thinks that if she selects a breakfast at random, it is likely that it will be oatmeal. Is she correct? Explain your reasoning.

_____

_____

_____

16. Draw Conclusions A researcher's garden contains 90 sweet pea plants, which have either white or purple flowers. About 70 of the plants have purple flowers, and about 20 have white flowers. Would you expect that one plant randomly selected from the garden will have purple or white flowers? Explain.

_____

_____

_____

17. The power goes out as Sandra is trying to get dressed. If she has 4 white T-shirts and 10 colored T-shirts in her drawer, is it likely that she will pick a colored T-shirt in the dark? What is the probability she will pick a colored T-shirt? Explain your answers.

_____

_____

18. James counts the hair colors of the 22 people in his class, including himself. He finds that there are 4 people with blonde hair, 8 people with brown hair, and 10 people with black hair. What is the probability that a randomly chosen student in the class does not have red hair? Explain.

_____

_____

19. Persevere in Problem Solving A bag contains 8 blue coins and 6 red coins. A coin is removed at random and replaced by three of the other color.

a. What is the probability that the removed coin is blue?

_____

b. If the coin removed is blue, what is the probability of drawing a red coin after three red coins are put in the bag to replace the blue one?

_____

c. If the coin removed is red, what is the probability of drawing a red coin after three blue coins are put in the bag to replace the red one?

_____

H.O.T.   **FOCUS ON HIGHER ORDER THINKING**

20. Draw Conclusions Give an example of an event in which all of the outcomes are not equally likely. Explain.

_____

_____

_____

21. Critique Reasoning A box contains 150 black pens and 50 red pens. Jose said the sum of the probability that a randomly selected pen will not be black and the probability that the pen will not be red is 1. Explain whether you agree.

_____

_____

_____

22. Communicate Mathematical Ideas A spinner has 7 identical sections. Two sections are blue, 1 is red, and 4 of the sections are green. Suppose the probability of an event happening is $\frac{2}{7}$. What does each number in the ratio represent? What outcome matches this probability?

_____

_____

Experimental Probability of Simple Events

? ESSENTIAL QUESTION

How do you find the experimental probability of a simple event?

EXPLORE ACTIVITY   COMMON CORE 7.SP.6, 7.SP.7b

Finding Experimental Probability

You can toss a paper cup to demonstrate *experimental probability*.

A Consider tossing a paper cup. Fill in the Outcome column of the table with the three different ways the cup could land.

B Toss a paper cup twenty times. Record your observations in the table.

Outcome	Number of Times

Reflect

1. Do the outcomes appear to be equally likely? _____

2. Describe the three outcomes using the words *likely* and *unlikely*.

_____

_____

3. Use the number of times each event occurred to approximate the probability of each event.

4. **Make a Prediction** What do you think would happen if you performed more trials?

_____

_____

_____

5. What is the sum of the probabilities in 3?

_____

Outcome	Experimental Probability
Open-end up	$\dfrac{\text{open-end up}}{20} = \dfrac{\Box}{20}$
Open-end down	$\dfrac{\text{open-end down}}{20} = \dfrac{\Box}{20}$
On its side	$\dfrac{\text{on its side}}{20} = \dfrac{\Box}{20}$

Calculating Experimental Probability

You can use *experimental probability* to approximate the probability of an event. An **experimental probability** of an event is found by comparing the number of times the event occurs to the total number of trials. When there is only one outcome for an event, it is called a **simple event**.

Experimental Probability

For a given experiment:

$$\text{Experimental probability} = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

EXAMPLE 1

Real World

COMMON CORE   7.SP.7b

Martin has a bag of marbles. He removed one marble at random, recorded the color and then placed it back in the bag. He repeated this process several times and recorded his results in the table. Find the experimental probability of drawing each color.

Color	Frequency
Red	12
Blue	10
Green	15
Yellow	13

STEP 1 Identify the number of trials: $12 + 10 + 15 + 13 = 50$

STEP 2 Complete the table of experimental probabilities. Write each answer as a fraction in simplest form.

Color	Experimental Probability
Red	$\frac{\text{frequency of the event}}{\text{total number of trials}} = \frac{12}{50} = \frac{6}{25}$
Blue	$\frac{\text{frequency of the event}}{\text{total number of trials}} = \frac{10}{50} = \frac{1}{5}$
Green	$\frac{\text{frequency of the event}}{\text{total number of trials}} = \frac{15}{50} = \frac{3}{10}$
Yellow	$\frac{\text{frequency of the event}}{\text{total number of trials}} = \frac{13}{50}$

> Substitute the results recorded in the table. You can also write each probability as a decimal or as a percent.

Reflect

6. **Communicate Mathematical Ideas** What are two different ways you could find the experimental probability of the event that Martin does **not** draw a red marble?

_____

_____

_____

_____

7. A spinner has three unequal sections: red, yellow, and blue. The table shows the results of Nolan's spins. Find the experimental probability of landing on each color. Write your answers in simplest form.

Color	Frequency
Red	10
Yellow	14
Blue	6

Math Talk
Mathematical Practices

Will everyone who does this experiment get the same results?

Making Predictions with Experimental Probability

A **simulation** is a model of an experiment that would be difficult or inconvenient to actually perform. You can use a simulation to find an experimental probability and make a prediction.

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EXAMPLE 2

COMMON CORE 7.SP.6

A baseball team has a batting average of 0.250 so far this season. This means that the team's players get hits in 25% of their chances at bat. Use a simulation to predict the number of hits the team's players will have in their next 34 chances at bat.

STEP 1 Choose a model.

Batting average $= 0.250 = \frac{250}{1,000} = \frac{1}{4}$

A standard deck of cards has four suits, hearts, diamonds, spades, and clubs. Since $\frac{1}{4}$ of the cards are hearts, you can let hearts represent a "hit." Diamonds, clubs, and spades then represent "no hit."

STEP 2 Perform the simulation.

Draw a card at random from the deck, record the result, and put the card back into the deck. Continue until you have drawn and replaced 34 cards in all.

(H = heart, D = diamond, C = club, S = spade)

H D D S H C H S D H C D C C D H H
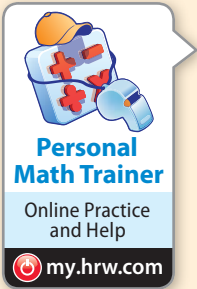S D D H C C H C H H D S S S C H D

Since the team has 34 chances at bat, you must draw a card 34 times.

STEP 3 Make a prediction.

Count the number of hearts in the simulation.

Since there are 11 hearts, you can predict that the team will have 11 hits in its next 34 chances at bat.

My Notes

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YOUR TURN

8. A toy machine has equal numbers of red, white, and blue foam balls which it releases at random. Ross wonders which color ball will be released next. Describe how you could use a standard number cube to predict the answer.

_____

_____

_____

_____

_____

Guided Practice

1. A spinner has four sections lettered A, B, C, and D. The table shows the results of several spins. Find the experimental probability of spinning each letter as a fraction in simplest form, a decimal, and a percent. **(Explore Activity and Example 1)**

Letter	A	B	C	D
Frequency	14	7	11	8

A: _____   B: _____

C: _____   D: _____

2. Rachel's free-throw average for basketball is 60%. She wants to predict how many times in the next 50 tries she will make a free throw. Describe how she could use 10 index cards to predict the answer. **(Example 2)**

_____

_____

_____

? ESSENTIAL QUESTION CHECK-IN

3. **Essential Question Follow Up** How do you find an experimental probability of a simple event?

_____

_____

12.2 Independent Practice

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4. Dree rolls a strike in 6 out of the 10 frames of bowling. What is the experimental probability that Dree will roll a strike in the first frame of the next game? Explain why a number cube would not be a good way to simulate this situation.

_____

_____

_____

5. To play a game, you spin a spinner like the one shown. You win if the arrow lands in one of the areas marked "WIN". Lee played this game many times and recorded her results. She won 8 times and lost 40 times. Use Lee's data to explain how to find the experimental probability of winning this game.

_____

_____

WIN

WIN

6. The names of the students in Mr. Hayes' math class are written on the board. Mr. Hayes writes each name on an index card and shuffles the cards. Each day he randomly draws a card, and the chosen student explains a math problem at the board. What is the probability that Ryan is chosen today? What is the probability that Ryan is **not** chosen today?

Anna	Alisha	Kenna	Bridget
Meghan	Cody	Parker	Grace
Michael	Gabe	Taylor	Joel
Kate	Kaylee	Shaw	Tessa
Jon	Ryan	Morgan	Leo

_____

7. Critique Reasoning A meteorologist reports an 80% chance of precipitation. Is this an example of experimental probability, written as a percent? Explain your reasoning.

_____

8. Mica and Joan are on the same softball team. Mica got 8 hits out of 48 times at bat, while Joan got 12 hits out of 40 times at bat. Who do you think is more likely to get a hit her next time at bat? Explain.

_____

_____

9. Make a Prediction In tennis, Gabby serves an ace, a ball that can't be returned, 4 out of the 10 times she serves. What is the experimental probability that Gabby will serve an ace in the first match of the next game? Make a prediction about how many aces Gabby will have for the next 40 serves. Justify your reasoning.

_____

10. Represent Real-World Problems Patricia finds that the experimental probability that her dog will want to go outside between 4 P.M. and 5 P.M. is $\frac{7}{12}$. About what percent of the time does her dog **not** want to go out between 4 P.M. and 5 P.M.?

_____

H.O.T. FOCUS ON HIGHER ORDER THINKING

11. Explain the Error Talia tossed a penny many times. She got 40 heads and 60 tails. She said the experimental probability of getting heads was $\frac{40}{60}$. Explain and correct her error.

_____

12. Communicate Mathematical Ideas A high school has 438 students, with about the same number of males as females. Describe a simulation to predict how many of the first 50 students who leave school at the end of the day are female.

_____

_____

_____

13. Critical Thinking For a scavenger hunt, Chessa put one coin in each of 10 small boxes. Four coins are quarters, 4 are dimes, and 2 are nickels. How could you simulate choosing one box at random? Would you use the same simulation if you planned to put these coins in your pocket and choose one? Explain your reasoning.

_____

_____

_____

_____

Experimental Probability of Compound Events

? ESSENTIAL QUESTION

How do you find the experimental probability of a compound event?

EXPLORE ACTIVITY

COMMON CORE 7.SP.8a, 7.SP.8b

Exploring Compound Probability

A **compound event** is an event that includes two or more simple events, such as flipping a coin *and* rolling a number cube. A compound event can include events that depend on each other or are independent. Events are independent if the occurrence of one event does not affect the probability of the other event, such as flipping a coin and rolling a number cube.

A What are the possible outcomes of flipping a coin once? _____

B What are the possible outcomes of rolling a standard number cube once? _____

C Complete the list for all possible outcomes for flipping a coin *and* rolling a number cube.

H1, H2, _____, _____, _____, _____, T1, _____, _____, _____, _____, _____

There are _____ possible outcomes for this compound event.

> H1 would mean the coin landed on heads, and the number cube showed a 1.

D Flip a coin and roll a number cube 50 times. Use tally marks to record your results in the table.

	1	2	3	4	5	6
H						
T						

E Based on your data, which compound event had the greatest experimental probability and what was it? The least experimental

probability? _____

F **Draw Conclusions** Did you expect to have the same probability for each possible combination of flips and rolls? Why or why not?

_____

_____

Calculating Experimental Probability of Compound Events

The experimental probability of a compound event can be found using recorded data.

A food trailer serves chicken and records the order size and sides on their orders, as shown in the table. What is the experimental probability that the next order is for 3-pieces with cole slaw?

	Green Salad	Macaroni & Cheese	French Fries	Cole Slaw
2 pieces	33	22	52	35
3 pieces	13	55	65	55

STEP 1 Find the total number of trials, or orders.

$$33 + 22 + 52 + 35 + 13 + 55 + 65 + 55 = 330$$

STEP 2 Find the number of orders that are for 3 pieces with cole slaw: 55.

STEP 3 Find the experimental probability.

$$P(3 \text{ piece} + \text{slaw}) = \frac{\text{number of 3 piece} + \text{slaw}}{\text{total number of orders}}$$

$$= \frac{55}{330} \quad \text{Substitute the values.}$$

$$= \frac{1}{6} \quad \text{Simplify.}$$

The experimental probability that the next order is for 3 pieces of chicken with cole slaw is $\frac{1}{6}$.

Math Talk

Mathematical Practices

Javier said the total number of orders is 8 and not 330. Is he correct? Explain.

YOUR TURN

1. Drink sales for an afternoon at the school carnival were recorded in the table. What is the experimental probability that the next drink is a small cocoa?

	Soda	Water	Cocoa
Small	77	98	60
Large	68	45	52

_____

Using a Simulation to Make a Prediction

You can use a simulation or model of an experiment to find the experimental probability of compound events.

EXAMPLE 2

COMMON CORE 7.SP.8c

At a street intersection, a vehicle is classified either as a *car* or a *truck*, and it can turn *left*, *right*, or go *straight*. About an equal number of cars and trucks go through the intersection and turn in each direction. Use a simulation to find the experimental probability that the next vehicle will be a car that turns right.
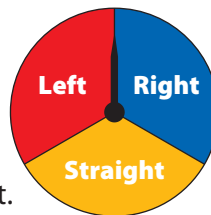
STEP 1 Choose a model.
Use a coin toss to model the two vehicle types.
Let Heads = **C**ar and Tails = **T**ruck

Use a spinner divided into 3 equal sectors to represent the *three* directions as shown.

Left Right
Straight

STEP 2 Find the sample space for the compound event.

There are 6 possible outcomes: **C**L, **C**R, **C**S, **T**L, **T**R, **T**S

STEP 3 Perform the simulation.

A coin was tossed and a spinner spun 50 times. The results are shown in the table.

	Car	Truck
Left	8	9
Right	6	11
Straight	9	7

STEP 4 Find the experimental probability that a car turns right.

$$P(\text{Car turns right}) = \frac{\text{frequency of compound event}}{\text{total number of trials}}$$
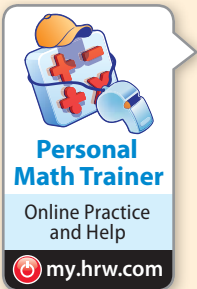
$$= \frac{6}{50} \quad \textit{Substitute the values.}$$

$$= \frac{3}{25} \quad \textit{Simplify.}$$

Based on the simulation, the experimental probability is $\frac{3}{25}$ that the next vehicle will be a car that turns right.

Reflect

2. **Make a Prediction** Predict the number of cars that turn right out of 100 vehicles that enter the intersection. Explain your reasoning.

_____

_____

YOUR TURN

3. A jeweler sells necklaces made in three sizes and two different metals. Use the data from a simulation to find the experimental probability that the next necklace sold is a 20-inch gold necklace.

	Silver	Gold
12 in.	12	22
16 in.	16	8
20 in.	5	12

_____

Guided Practice

1. A dentist has 400 male and female patients that range in ages from 10 years old to 50 years old and up as shown in the table. What is the experimental probability that the next patient will be female and in the age range 22–39? (Explore Activity and Example 1)

	Range: 10–21	Range: 22–39	Range: 40–50	Range: 50+
Male	44	66	32	53
Female	36	50	45	74

_____

2. At a car wash, customers can choose the type of wash and whether to use the interior vacuum. Customers are equally likely to choose each type of wash and whether to use the vacuum. Use a simulation to find the experimental probability that the next customer purchases a deluxe wash and no interior vacuum. Describe your simulation. (Example 2)
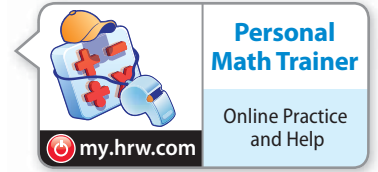
_____

_____

_____

CAR WASH

Standard ⬤
Deluxe ⬤
Superior ⬤

Vacuum
⬤ yes   ⬤ no

? ESSENTIAL QUESTION CHECK-IN

3. How do you find the experimental probability of a compound event?

_____

_____

12.3 Independent Practice

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4. **Represent Real-World Problems** For the same food trailer mentioned in Example 1, explain how to find the experimental probability that the next order is two pieces of chicken with a green salad.

_____

_____

The school store sells spiral notebooks in four colors and three different sizes. The table shows the sales by size and color for 400 notebooks.

	Red	Green	Blue	Yellow
100 Pages	55	37	26	12
150 Pages	60	44	57	27
200 Pages	23	19	21	19

5. What is the experimental probability that the next customer buys a red notebook with 150 pages?

_____

6. What is the experimental probability that the next customer buys any red notebooks?

_____

7. **Analyze Relationships** How many possible combined page count and color choices are possible? How does this number relate to the number of page size choices and to the number of color choices?

_____

_____

A middle school English teacher polled random students about how many pages of a book they read per week.

	6th	7th	8th
75 Pages	24	18	22
100 Pages	22	32	24
150 Pages	30	53	25

8. **Critique Reasoning** Jennie says the experimental probability that a 7th grade student reads at least 100 pages per week is $\frac{16}{125}$. What is her error and the correct experimental probability?

_____

9. **Analyze Relationships** Based on the data, which group(s) of students should be encouraged to read more? Explain your reasoning.

_____

_____

10. Make a Conjecture Would you expect the probability for the simple event "rolling a 6" to be greater than or less than the probability of the compound event "rolling a 6 and getting heads on a coin"? Explain.

_____

_____

11. Critique Reasoning Donald says he uses a standard number cube for simulations that involve 2, 3, or 6 equal outcomes. Explain how Donald can do this.

_____

_____

_____

12. Draw Conclusions Data collected in a mall recorded the shoe styles worn by 150 male and for 150 female customers. What is the probability that the next customer is male and has an open-toe shoe (such as a sandal)? What is the probability that the next male customer

	Male	Female
Open toe	11	92
Closed toe	139	58

has an open-toe shoe? Are the two probabilities the same? Explain.

_____

_____

_____

13. What If? Suppose you wanted to perform a simulation to model the shoe style data shown in the table. Could you use two coins? Explain.

_____

_____

14. Represent Real-World Problems A middle school is made up of grades 6, 7, and 8, and has about the same number of male and female students in each grade. Explain how to use a simulation to find the experimental probability that the first 50 students who arrive at school are male and 7th graders.

_____

_____

_____

_____

Making Predictions with Experimental Probability

? **ESSENTIAL QUESTION**

How do you make predictions using experimental probability?

Using Experimental Probability to Make a Prediction

Scientists study data to make predictions. You can use probabilities to make predictions in your daily life.

Math On the Spot
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EXAMPLE 1   Real World

COMMON CORE 7.SP.6

Danae found that the experimental probability of her making a bull's-eye when throwing darts is $\frac{2}{10}$, or 20%. Out of 75 throws, about how many bull's-eyes could she predict she would make?

Method 1: Use a proportion.

$\frac{2}{10} = \frac{x}{75}$ *Write a proportion. 2 out of 10 is how many out of 75?*

$\frac{2}{10} = \frac{x}{75}$
$\times 7.5$

$\frac{2}{10} = \frac{15}{75}$ *Since 10 times 7.5 is 75, multiply 2 times 7.5 to find the value of x.*
$\times 7.5$

$x = 15$

Method 2: Use a percent equation.

$0.20 \cdot 75 = x$ *Find 20% of 75.*

$15 = x$

You can write probabilities as ratios, decimals, or percents.

Danae can predict that she will make about 15 bull's-eye throws out of 75.

YOUR TURN

1. A car rental company sells accident insurance to 24% of its customers. Out of 550 customers, about how many customers are predicted to

   purchase insurance? _____

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Using Experimental Probability to Make a Qualitative Prediction

A prediction is something you reasonably expect to happen in the future. A qualitative prediction helps you decide which situation is more likely in general.

EXAMPLE 2

Real World

COMMON CORE 7.SP.6

A doctor's office records data and concludes that, on average, 11% of patients call to reschedule their appointments per week. The office manager predicts that 23 appointments will be rescheduled out of the 240 total appointments during next week. Explain whether the prediction is reasonable.

Method 1: Use a proportion.

$$\frac{11}{100} = \frac{x}{240}$$ Write a proportion. 11 out of 100 is how many out of 240?

$$\frac{11}{100} = \frac{x}{240}$$
    $\times 2.4$

    $\times 2.4$
$$\frac{11}{100} = \frac{26.4}{240}$$ Since 100 times 2.4 is 240, multiply 11 times 2.4 to find the value of x.
    $\times 2.4$

$$x = 26.4$$

Math Talk

Mathematical Practices

Does 26.4 make sense for the number of patients?

26.4 is the average number of patients that would call to reschedule.

Method 2: Use a percent equation.

$$0.11 \cdot 240 = x$$ Find 11% of 240.

$$26.4 = x$$ Solve for x.

The prediction of 23 is reasonable but a little low, because 23 is a little less than 26.4.

YOUR TURN

2. In emails to monthly readers of a newsletter 3% of the emails come back undelivered. The editor predicts that if he sends out 12,372 emails, he will receive 437 notices for undelivered email. Do you agree with his prediction?

   Explain. _____

   _____

   _____

Making a Quantitative Prediction

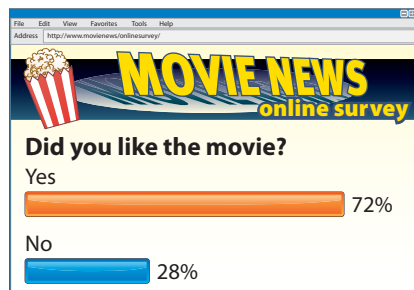You can use proportional reasoning to make quantitative predictions and compare options in real-world situations.

EXAMPLE 3 *Problem Solving*

An online poll for a movie site shows its polling results for a new movie. If a newspaper surveys 150 people leaving the movie, how many people can it predict will like the movie based on the online poll? Is the movie site's claim accurate if the newspaper has 104 people say they like the movie?

File Edit View Favorites Tools Help
Address http://www.movienews.com/onlinesurvey/

MOVIE NEWS
online survey

Did you like the movie?
Yes
72%
No
28%

Analyze Information

The **answer** is a prediction for how many people out of 150 will like the movie based on the online poll. Also tell whether the 104 people that say they like the movie is enough to support the movie site's claim.

List the important information:
- The online poll says 72% of movie goers like the new movie.
- A newspaper surveys 150 people.

Formulate a Plan

Use a proportion to calculate 72% of the 150 people surveyed.

Solve

$$\frac{72}{100} = \frac{x}{150}$$

Set up a proportion. 72 out of 100 is how many out of 150?

$$\frac{72}{100} = \frac{x}{150}$$ ↘ ×1.5 ↗

↗ ×1.5 ↘
$$\frac{72}{100} = \frac{108}{150}$$ ↘ ×1.5 ↗

Since 100 times 1.5 is 150, multiply 72 times 1.5 to find the value of x.
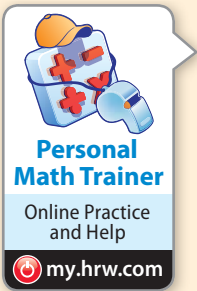
$$x = 108$$

The newspaper can predict that 108 out of 150 people will say they like the movie, based on the online poll.

Justify and Evaluate

Since 108 is close to 104, the newspaper survey and the online poll show that about the same percent of people like the movie.

My Notes

YOUR TURN

3. On average, 24% of customers who buy shoes in a particular store buy two or more pairs. One weekend, 350 customers purchased shoes. How many can be predicted to buy two or more pairs? If 107 customers buy more than two pairs, did more customers than normal buy two or more pairs?

_____

_____

Guided Practice

1. A baseball player reaches first base 30% of the times he is at bat. Out of 50 times at bat, about how many times will the player reach first base? (Example 1)

_____

2. The experimental probability that it will rain on any given day in Houston, Texas, is about 15%. Out of 365 days, about how many days can residents predict rain? (Example 1)

_____

3. A catalog store has 6% of its orders returned for a refund. The owner predicts that a new candle will have 812 returns out of the 16,824 sold. Do you agree with this prediction? Explain. (Example 2)

_____

_____

4. On a toy assembly line, 3% of the toys are found to be defective. The quality control officer predicts that 872 toys will be found defective out of 24,850 toys made. Do you agree with this prediction? Explain. (Example 2)

_____

_____

5. A light-rail service claims to be on time 98% of the time. Jeanette takes the light-rail 40 times one month, how many times can she predict she will be on time? Is the light-rail's claim accurate if she is late 6 times? (Example 3)

_____

_____

6. On average, a college claims to accept 18% of its applicants. If the college has 5,000 applicants, predict how many will be accepted. If 885 applicants are accepted, is the college's claim accurate? (Example 3)

_____

_____

❓ ESSENTIAL QUESTION CHECK-IN

7. How do you make predictions using experimental probability?

_____

_____

_____

12.4 Independent Practice

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The table shows the number of students in a middle school at the beginning of the year and the percentage that can be expected to move out of the area by the end of the year.

	6th	7th	8th
Number of Students	250	200	150
% Moves	2%	4%	8%

8. How many 7th grade students are expected to move by the end of the year? If 12 students actually moved, did more or fewer 7th grade students move than expected? Justify your answer.

_____

_____

9. Critique Reasoning The middle school will lose some of its funding if 50 or more students move away in any year. The principal claims he only loses about 30 students a year. Do the values in the table support his claim? Explain.

_____

_____

_____

10. Represent Real-World Problems An airline knows that, on average, the probability that a passenger will not show up for a flight is 6%. If an airplane is fully booked and holds 300 passengers, how many seats are expected to be empty? If the airline overbooked the flight by 10 passengers, about how many passengers are expected to show up for the flight? Justify your answer.

_____

_____

_____

11. Draw Conclusions In a doctor's office, an average of 94% of the clients pay on the day of the appointment. If the office has 600 clients per month, how many are expected not to pay on the day of the appointment? If 40 clients do not pay on the day of their appointment in a month, did more or fewer than the average not pay?

_____

12. Counterexamples The soccer coach claimed that, on average, only 80% of the team come to practice each day. The table shows the number of students that came to practice for 8 days. If the team has 20 members, how many team members should come to practice to uphold the coach's claim? Was the coach's claim accurate? Explain your reasoning.

	1	2	3	4	5	6	7	8
Number of Students	18	15	18	17	17	19	20	20

_____

_____

13. What's the Error? Ronnie misses the school bus 1 out of every 30 school days. He sets up the proportion $\frac{1}{30} = \frac{180}{x}$ to predict how many days he will miss the bus in the 180-day school year. What is Ronnie's error?

_____

 FOCUS ON HIGHER ORDER THINKING

14. Persevere in Problem Solving A gas pump machine rejects 12% of credit card transactions. If this is twice the normal rejection rate for a normal gas pump, how many out of 500 credit cards transactions would a

normal gas pump machine reject? _____

15. Make Predictions An airline's weekly flight data showed a 98% probability of being on time. If this airline has 15,000 flights in a year, how many flights would you predict to arrive on time? Explain whether you can use the data to predict whether a specific flight with this airline will be on time.

_____

_____

_____

16. Draw Conclusions An average response rate for a marketing letter is 4%, meaning that 4% of the people who receive the letter respond to it. A company writes a new type of marketing letter, sends out 2,400 of them, and gets 65 responses. Explain whether the new type of letter would be considered to be a success.
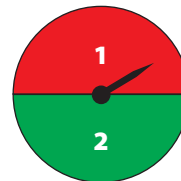
_____

_____

_____

Ready to Go On?

12.1 Probability

1. Josue tosses a coin and spins the spinner at the right. What are all the possible outcomes?

_____

12.2 Experimental Probability of Simple Events

2. While bowling with friends, Brandy rolls a strike in 6 out of 10 frames. What is the experimental probability that Brandy will roll a strike in the first frame of the next game?

_____

3. Ben is greeting customers at a music store. Of the first 20 people he sees enter the store, 13 are wearing jackets and 7 are not. What is the experimental probability that the next person to enter the store will be wearing a jacket?

_____

12.3 Experimental Probability of Compound Events

4. Auden rolled two number cubes and recorded the results.

Roll #1	Roll #2	Roll #3	Roll #4	Roll #5	Roll #6	Roll #7
2, 1	4, 5	3, 2	2, 2	1, 3	6, 2	5, 3

What is the experimental probability that the sum of the next two numbers rolled is greater than 5?
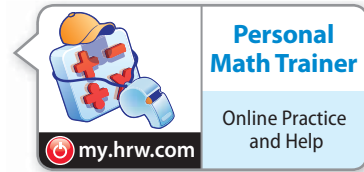
_____

12.4 Making Predictions with Experimental Probability

5. A player on a school baseball team reaches first base $\frac{3}{10}$ of the time he is at bat. Out of 80 times at bat, about how many times would you predict he will reach first base?

_____

? ESSENTIAL QUESTION

6. How is experimental probability used to make predictions?

_____

_____

_____

MODULE 12
MIXED REVIEW
Assessment Readiness

COMMON CORE

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1. Consider each addition problem. Is the sum positive?
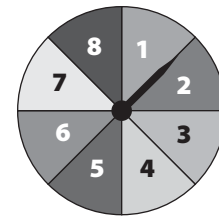
   Select Yes or No for A–C.

   A. $-18 + 27 + (-6)$ ⭘ Yes ⭘ No

   B. $12 + (-19) + 5$ ⭘ Yes ⭘ No

   C. $32 + (-24) + (-6)$ ⭘ Yes ⭘ No

2. The spinner at right has 8 equal sections, as shown.

   Choose True or False for each statement.

   A. The probability of landing on ⭘ True ⭘ False
   an odd number is $\frac{1}{2}$.

   B. The probability of not landing ⭘ True ⭘ False
   on 6 is $\frac{7}{8}$.

   C. The probability of landing on a ⭘ True ⭘ False
   number less than 3 is $\frac{3}{8}$.

3. The table shows the sizes and colors of T-shirts sold at a store in the past week. To the nearest percent, what is the experimental probability that the next customer who buys a T-shirt at the store will buy a large blue T-shirt? Explain how you solved this problem.

	Small	Medium	Large
White	32	49	28
Blue	12	36	20
Black	20	42	23

4. The experimental probability that a delivery service will deliver a package late is 3%. An employee at the delivery service predicts that 45 out of 15,000 packages will be delivered late. Is this prediction reasonable? Justify your answer.